STATIONARY ROTATION STABILITY OF UNBALANCED ROTOR WITH AUTOBALANCING DEVICE WITH LIQUID ON FLEXIBLE SHAFT

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The condition of rotor rotation stability with liquid autabalancing device consisting of chamber, float and incompressible homogeneous liquid filling the space between them has been obtained. The restoring force and forces of internal and external friction take effect on the rotor. The latter depend linearly respectively on strain rate and absolute velocity of rotor connection point to the shaft.

To remove rotating bodies unbalance various autabalancing device with liquid (ADL) are used [1]. When using such systems it is necessary to know critical angular velocities at which stationary rotations stability is disturbed. In a number of papers, for example in [2, 3], the approximate conditions of stability of balanced cylinder steady rotation filled partially with liquid which are difficult to be applied to the systems of ADL are obtained. The immediate investigation of stability of rotors rotation with liquid ADL is not described in literature.

In the proposed paper by analogy with [4] for unbalanced discs at flexible shaft the stability of rotor rotation with liquid ADL without free surface when influenced by forces of internal and external friction is studied. The nature of these forces is stated in detail in [4]. So the forces of external friction are caused by viscous resistance of environment, supports, special dampers and depend on rates of absolute displacements of rotor and shaft points; forces of internal friction are caused by material particles resistance and are taken as proportional to the shaft strain rate to a first approximation. The investigation of the considered forces ratio influence on stability of rotor rotation with ADL is of some interest. The solution of such problem for the unbalanced disc without ADL is given in [4]. Let rotor with ADL is fixed symmetrically with respect to the supports of vertical flexible shaft passing through its geometric centre $O_1$ (Figure).

ADL consists of the correction chamber — 1 with the height $h$, the float — 2 and homogeneous incompressible fluid — 3 filling the space between their walls. Centre of rotor mass (point $P$) is shifted from $O_1$ to the distance $O_1P=e$. Point is a projection of shift support axis to the plane of motion. At system rotation the float sags at a place of rotor attaching point per the value $O_1O_2$, the float for which geometric and material symmetry axes coincide as well in floating gyroscopes [5] is centered on rotation axis $O_1$ due to pressure forces and fluid flows to the side of sag. Let us suppose that at rotor disturbed motion fluid separation from walls does not occur and float centering remains. In this case fluid layer centre of mass is on the line of $O_1O_2$ centers in point $G$. The stated suppositions allow mapping out a hydrodynamic problem.

According to [4] let us introduce in a plane of motion of points $O_1$, $G$, $P$ two coordinate systems with common origin in point $O_2$ fixed $O_2xy$ and mobile $O_2\xi\eta$, axis $O_2\xi$ which is parallel to the segment $O_2P$.

Laws of angular motion of rotor and system $O_2\xi\eta$ are determined by the same rotation angle $\beta(t)$ ($t$— time) therefore, rotor in the mobile coordinate system can move only steadily. Let us take the coordinates in a fixed system of point $O_1$ which are denoted by $x$, $y$ as generalized coordinates. We consider that rotor is influenced from the side of a shaft by elastic force $F_1=-cO_1O_2$, forces of internal and external friction $F_2=-kV_2$, and $F_3=-\chi V_3$, applied in the point $O_3$ proportional correspondingly to a shaft bending $O_1O_3=x\cdot i+y\cdot j$; rate of point $O_3$ in the mobile coordinate system $O_2\xi\eta$

$$F_1'= (x\cdot y\cdot \beta)\, \tau + (y-x\cdot \beta)\, \mathbf{j}$$

and absolute velocity

$$F_2'=\dot{x}\cdot \tau + \dot{y}\cdot \mathbf{j},$$

where $\mathbf{i}$, $\mathbf{j}$ are the orts of rectangular coordinate system $O_2xy$. Equilibrium condition of the enumerated forces and system inertial forces is written down in the form of

$$-cO_1O_3-\chi V_3 XO_3-\beta\mathbf{r}_p-m\mathbf{a}_p-m\mathbf{a}_G=0.$$  

Therein $m_1$, $m_2$ and $\beta$, $\alpha$ are masses and acceleration of mass centers of rotor and liquid respectively. The latter are calculated through the coordinates of points $P$ and $G$ in the fixed reference system.

$$x_p=x+e\cdot \cos \beta, \quad y_p=y+e\cdot \sin \beta,$$

$$x_G=r\cdot x, \quad y_G=r\cdot y, \quad \beta(t)$$

(4)
where \( r=r_1/(r_1^2-r_2^2) \), \( r_1 \) and \( r_2 \) are the radii of the chamber and the float respectively. Projecting (3) on the axes \( x, y \) subject to (1, 2) and (4) we obtain the differential equations of motion of rotor with ADL

\[
\begin{align*}
mx'' + (p + \chi)x' + cx + pm\beta = mE(\beta^2 \cos \beta + \beta \sin \beta); \\
my'' + (p + \chi)y' + cy - px\beta = mE(\beta^2 \sin \beta - \beta \cos \beta),
\end{align*}
\]

in which the notation \( m=m_1+r m_2 \) is entered.

Supposing in (5) that \( \epsilon=0, m_2=0 \) and \( \epsilon=0 \) we have the equation of motion of balanced disk mass centre on an elastic shaft at internal friction influence given in [6]. Comparing the equations in [6] with (5) we correlate the ratio between the coefficient \( p \) and coefficient \( k \) entering into dependence of stress on deformation of shaft viscoelastic material \( \sigma=E\varepsilon+\kappa\dot{\varepsilon} \). Therein there is shaft flexural stiffness for simply supported shaft with the length \( l \) with a disk in the middle [6] \( c=48EL/l^3 \); \( E \) is the modulus of material elasticity; \( I \) is the inertia moment of sectional area relative to the centroidal axis of shaft bending perpendicular plane. At \( \beta=\omega=\text{const} \) of the equation (5) the calculation is admitted

\[
x_\epsilon = A \cos(\omega t + \gamma); \quad y_\epsilon = A \sin(\omega t + \gamma);
\]

\[
A = \frac{mE\omega_0^2}{\sqrt{(c-m\omega^2)^2 + \chi^2\omega^4}}; \quad \tan \gamma = -\frac{\chi \omega^2}{c-m\omega^2};
\]

where \( A \) is the shaft bending, \( \gamma \) is the motion phase angle (the angle between rotor unbalance vector \( O, P \) and bending deflection \( O, Q \)).

It is seen from (6) that the given solution corresponds to the stationary rotation of rotor with FBD does not depend on the forces of internal friction. It is obvious as the shaft deformation does not change at such motion. The calculation (6) is studied in detail in [7].

To study the stability of the rotation under consideration let us enter the deviations \( x'=x-x_\epsilon, y'=y-y_\epsilon \). Supposing that angular velocity \( \omega \) remains constant the disturbed motion equation is obtained from (5) subject to (6).

\[
\begin{align*}
mx'' + (p + \chi)x' + cx' + pm\gamma' = 0; \\
my'' + (p + \chi)y' + cy' - px\gamma' = 0.
\end{align*}
\]

Characteristic equation for the system (7) has the form

\[
a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0,
\]

where \( a_0 = m^2, \quad a_1 = 2m(p + \chi), \quad a_2 = (p + \chi)^2 + 2cm, \quad a_3 = 2c(p + \chi), \quad a_4 = c^2 + \chi^2 \omega^2 \).

Gurvits matrix \( H \) of the coefficients of equation (8) according to [8] is written

\[
H = \begin{bmatrix} a_1 & a_2 & 0 & 0 \\ a_0 & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & 0 \\ 0 & 0 & a_0 & a_4 \end{bmatrix}.
\]

The conditions of stability are of the form of inequations [8]

\[
\Delta > 0; \quad i=1...4,
\]

where \( \Delta \) are the main diagonal minors of matrix \( H \).

Inequations (9) are fulfilled if

\[
\omega < \omega_0 = (1 + \frac{2}{\mu}) \sqrt{\frac{c}{m_1}} \sqrt{\frac{1}{\mu}}, \quad \mu = \frac{m}{m_1} > 1.
\]

At \( \mu=1 \) we obtain the condition of stability of rotor stationary rotation without ADL given in [4]. Comparing (10) with a congruent inequation in [4] we conclude that the limiting value of angular velocity at stationary rotation of rotor with ADL \( \omega_0(\mu) \) is less than for the rotor without ADL. It is shown in [7] that fluid ADL decreases shaft bending and system unbalance at angular velocities

\[
\omega > \omega_* = \sqrt{\frac{c}{m_1}} \sqrt{\frac{2}{1+\mu}}.
\]

It follows from (10, 11) that steady stationary rotation of rotor with fluid ADL decreasing vibration is possible at angular velocities meeting the condition \( \omega_0 < \omega < \omega_* \).

Conclusions

Autobalancing device with liquid connected to the rotor decreases angular critical speed of the shaft the transition through which results in system rotation instability. Stable system operation is realized in the range of angle velocities determined by the parameters of rotor and autobalancing device.

REFERENCES


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